Corrections for Nonlinear Vector Network Analyzer Measurements Using a Stochastic Multi-Line/Reflect Method

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Abstract — A new 16-term statistical calibration has been developed for the correction of vector network analyzer (VNA) data. The method uses multiple measurements of generic transmission line and reflection standards. Using a functional model of the system and transmission line standards, we apply a nonlinear least-squares estimator to simultaneously optimize the correction terms in the measurement model *and* the propagation constant. The method provides estimates of the uncertainty on each of the parameters using the final Jacobian. This paper shows for the first time an application of the new calibration to a commercial nonlinear VNA, plus quantitative statements regarding the quality of the parameters.

Index Terms — Measurements, Calibrations, Vector Network Analyzer, Propagation Constant.

I. INTRODUCTION

In this paper, we apply a newly-developed statistical calibration technique to data acquired with a commercial nonlinear vector network analyzer (NVNA). The method uses a 16-term scattering-parameter model[1] and a convenient nonlinear-least-squares optimization process, that is easily implemented using readily available software tools. Using multiple measurements of uniform transmission line and reflection standards, we minimize the residuals in a semi-linear system of equations derived from noisy measurements. We attain estimates for each of the 15 unique coefficients in the two-port correction model while concurrently optimizing the transmission-line propagation constant γ, the key parameter in the model descriptions of transmission-line standards.

The need for sound measurement assurance is the prime motivator for this work. This new method provides estimates for the uncertainty in all parameters along with a comparative test for success which represent the salient features of the current work. The microwave measurement community is looking more and more to statistical methods [1-5] in order to improve scattering-parameter (*S*-parameter) measurements beyond the tradition of closed-form, deterministic approaches. We are not only improving the *S*-parameter corrections this way, but we are able to determine quantitative measures on the quality of corrected data.

While it is true that commercial vector network analyzers (VNAs) can measure the *S*-parameters of microwave circuits with a high degree of precision, including cross-channel coupling of less than -60 dB, the need for full 16-term VNA calibrations is driven by a set of specific applications and metrology lab requirements. One application is the calibration of wide-band network analyzers used for nonlinear circuit characterization. Out of necessity, these instruments provide wide-bandwidth (> 4 MHz) measurement channels resulting in an increase in measurement noise of more than three orders of magnitude above conventional VNAs. Another application is on-wafer millimeter-wave measurements where probe-to-probe coupling can exceed -40 dB. Of interest to the authors is the need to quantify the quality of data in metrology-grade laboratories. These and most R&D laboratory applications will benefit significantly from statistical estimators and methods that can track measurand variability with changes in noise, model equations, and measurement conditions.

In the following sections of this paper, we briefly describe a straightforward 16-term estimator and demonstrate its application to an NVNA. We present new results including estimates of uncertainty in the error-correction parameters.

II. CALIBRATION METHOD

The proposed method extends the stochastic calibration framework of Van Hamme and Vanden Bossche [1] and Van Moer *et. al.* [2,3] by applying a statistical, least-squares estimation to the Multiline TRL problem [6]. Here, we consider a general mixture of standards where some are described by wave propagation along uniform two-conductor waveguides. The desire to mix standard types is similar to the goals of Van Moer and Rolain [2] and Williams *et. al.* [4,5]; we report here on complete 16-term solutions and distinct estimation methods.

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[1] We refer to the 16-term two-port *S*-parameter model, recognizing that the rank of such a correction matrix is no greater than 15.

The method treats the two-port VNA case generically, without associating the terms with specific physical features of the instrument. The 16-term, complex-valued correction model is defined as:

$$\begin{bmatrix} b_1 \\ b_2 \\ a_1 \\ a_2 \end{bmatrix}^V = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \theta_{14} \\ \theta_{21} & \theta_{22} & \theta_{23} & \theta_{24} \\ \theta_{31} & \theta_{32} & \theta_{33} & \theta_{34} \\ \theta_{41} & \theta_{42} & \theta_{43} & \theta_{44} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ a_1 \\ a_2 \end{bmatrix}^D, \quad (1)$$

where a and b are the complex wave amplitudes, the superscript D indicates the wave-variables at the device-under test (DUT) reference planes, and the superscript V denotes the ideal noise-free data inside the VNA.

While (1) uses wave-variable data accessible to NVNAs, we identify θ here as the system that links the instrument-transformed S-parameters to the actual S-parameters of the DUT. Expressing the relation $b = Sa$, we can rewrite the DUT to VNA model as

$$S^V = \begin{bmatrix} \Theta_{11} & \Theta_{12} \end{bmatrix} \begin{bmatrix} S^D \\ I_2 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \Theta_{21} & \Theta_{22} \end{bmatrix} \begin{bmatrix} S^D \\ I_2 \end{bmatrix} \end{bmatrix}^{-1}, \quad (2)$$

where I_2 is the 2x2 block identity matrix, and Θ_{xy} are the four block matrices of θ in (1). This is the functional model for the two-port system which is taken to be exact in the absence of noise.

The first goal of the current statistical method is to identify a quasilinear model f where measurement noise in S will enter linearly. This is one of the key advantages of this calibration method. For this purpose, we will use multiple transmission-line standards defined in terms of the propagation constant γ. In the presence of an additive noise perturbation, we reformulate the model for S^V as

$$S^M = f\left[\theta, S^D(\gamma)\right] + e_n, \quad (3)$$

where S^M now represents an individual measurement with complex noise e_n. The S^M are formulated in terms of the measured a and b vectors in forward and reverse instrument states following Van Hamme and Vanden Bossche [1].

The deterministic model equations for the transmission line and offset reflection standards are written in terms of γ. Lumped-element standards can also be used, but they require their own frequency-dependent models.

Considering S-parameters defined in terms of the transmission-line characteristic impedance Z_c, the line model is represented by

$$S_L^D = \begin{bmatrix} 0 & e^{-\gamma\ell} \\ e^{-\gamma\ell} & 0 \end{bmatrix}, \quad (4)$$

where ℓ is the distance between the measurement reference planes. For offset reflections, the defined S are given by

$$S_R^D = \begin{bmatrix} \Gamma_R e^{-2\gamma\ell} & 0 \\ 0 & \Gamma_R e^{-2\gamma\ell} \end{bmatrix}, \quad (5)$$

where ℓ is the distance between the reflection element and the nominal reference plane location, and Γ_R is the reflection coefficient of the termination ($\Gamma_R = -1$ for an ideal short circuit).

Using Kronecker multiplication defined by Brewer [7], we can rewrite the functional model in (2) as the basis of a system of equations,

$$\left\{ \begin{bmatrix} (S^D)^\top & I_2 \end{bmatrix} \otimes \begin{bmatrix} -I_2 & S^M \end{bmatrix} \right\} \mathrm{vec}\{\theta\} = 0, \quad (6)$$

where the *vec* operator forms a vector by stacking subsequent columns of θ on top of each other.

For each standard, there will be four rows (equations) in the system. In practice, we perform repeated measurements of each standard in the calibration kit, making at least six connector repeats. Instead of introducing four new rows for each repeat, we take the mean value of the measurements and the covariance of the mean into the estimator.

Because we are estimating model parameters in the least-squares sense, we cast the equation residuals as

$$e = y - H(\gamma)\theta_v. \quad (7)$$

Here, the matrix H is a collection of measurement and modeling terms from (6) less one column of measurements -y, that gets moved when normalizing the model with $\theta_{44} = +1$.

Note that residual errors in (7) are nonlinear in the parameters when we include γ as a parameter. The method applies a nonlinear least-squares minimization of a weighted cost function L_{kw} that is quadratic in the measurements. It minimizes

$$L_{kw} = e^\mathsf{H} C_e^{-1} e, \quad (8)$$

where H denotes the Hermitian, or complex conjugate transpose, and C_e is the covariance in e due to measurement noise.

Since there is no *a priori* assurance that the calibration problem will involve a globally convex cost function, we devote careful attention to acquiring good starting values for the 15 correction terms *plus* the propagation constant γ of the transmission-line standards.

The starting value for the propagation constant is derived from the uncorrected standard measurements and a modification of the Multiline method [6] that avoids problems associated with root selection. With this starting

value for γ, we define the S^D of each transmission line standard and use a linear-least-squares estimator to generate the starting values for $\boldsymbol{\theta}$.

With these excellent starting values, we apply a Levenberg-Marquardt modification of the Gauss-Newton method following procedures described in [8], and optimize the parameters at each frequency point independently. To approach a minimum variance estimate, the method uses the inverse of the measurement noise covariance C_m in weighting the equation residuals. In the absence of the exact measurement noise covariance matrix, the method uses an estimate $C_{m,est}$ obtained from the repeated measurements of each standard. Typically, these repeated data include both electrical measurement noise and the contact variance.

At the end of the iterative optimization process, the method produces estimates of parameters $\boldsymbol{\theta}_{est}$ and γ_{est}. It then estimates a parameter uncertainty using the final Jacobian matrix to represent the residual variability of these parameters about their optimized values.

III. APPLICATION TO NVNA DATA

Using extensive simulations of random measurement noise of the same covariance as that observed in our repeated measurements for each standard, and assuming an ideal VNA model, we demonstrated the validity of the method by recovering the expected parameter estimates and the expected cost-function distribution. Comparisons of the propagation constant data with the Multiline method showed differences of less than 0.4% over the frequency band of 1–40 GHz.

With quantifiable confidence in the method, we applied it to a nonlinear vector network analyzer. Here, we collected uncorrected $\boldsymbol{a}$ and $\boldsymbol{b}$ vector measurements for 6 repeated connections to each of the 6 line and 2 reflect standards defined in Table I. We formulated the mean S^M and covariance values and ran the optimization software on these data.

TABLE I

Definition of Line & Reflect Standards

Coplanar Waveguides, Au Metalization on Alumina

Standard	Length/Offset (mm)
Line 1	0
Line 2	0.5
Line 3	2.135
Line 4	3.2
Line 5	6.565
Line 6	19.695
Reflect 1	0/0
Reflect 2	0/1.5

The data in Fig. 1 show the estimated value of the propagation constant expressed as the real part of the effective relative permittivity ($\varepsilon_{r,eff} = -\gamma^2 c^2/\omega^2$). We also compared these data to values obtained from VNA measurements of the same standards using the NIST Multiline method, and plotted the difference between the two methods in Fig. 1. The two methods agree well from just under 1 GHz to 40 GHz. The significance of the increased difference at low frequencies is yet to be explored.

Figure 2 gives an estimate of the uncertainty in γ_{est} as a function of frequency. Since the propagation constant is a complex function, we report the standard deviation σ to be that of a circularly Gaussian-distributed error about the estimate. This gives the standard deviation of $|\gamma_{est}|$. Above 6 GHz, we see a frequency behavior that follows the observed trends in the measurement noise covariance, and also note increased uncertainty at low frequencies due to the limited phase difference information in finite line lengths.

The estimates of all 15 correction coefficients are plotted in Fig. 3; estimates of their uncertainty are shown in Fig. 4. The upper three curves in Fig. 3 represent the on-diagonal $\theta_{i=j}$ parameters, and the middle band show terms related to the instrument mismatch, while the lower band of curves show the cross-talk terms.

The uncertainty in the correction parameters is again represented as the σ of a circularly Gaussian error in Fig. 4, showing that uncertainty increases with frequency. Figures 3 and 4 reveal an important point: ignoring cross-talk terms in the correction model may introduce errors that are bigger than the level of measurement uncertainty attainable in good calibration methods.
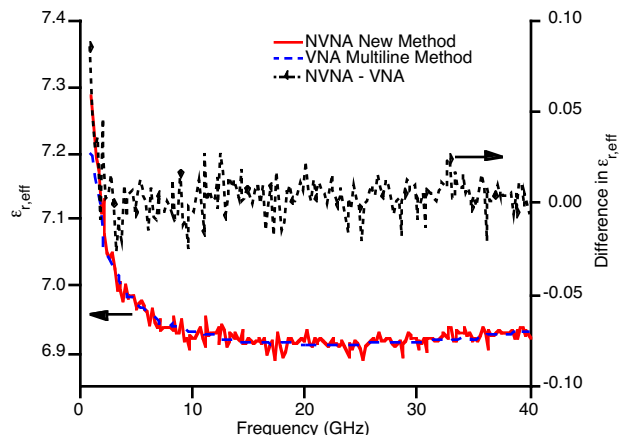

Fig. 1. Estimated propagation-constant parameter of transmission-line standards from NVNA data compared to VNA results acquired with the Multiline method. Data are shown as real part of effective relative permittivity $\varepsilon_{r,eff}$.
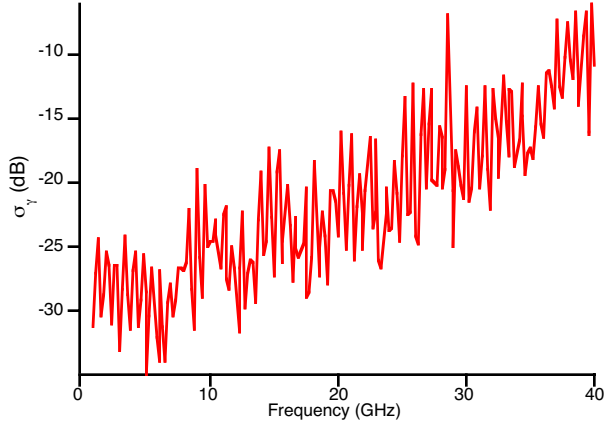
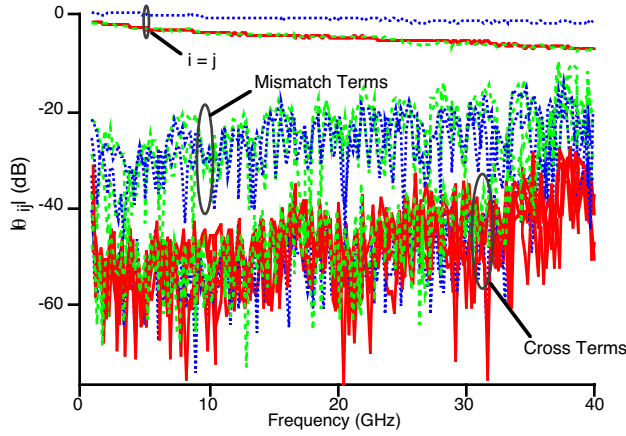Fig. 2. Estimate of the uncertainty in $|\gamma_{est}|$.

Fig. 3. Nonlinear least-squares estimate $\boldsymbol{\theta}_{est}$: upper curves give $\theta_{i=j}$; middle curves give θ_{13}, θ_{31}, θ_{24}, and θ_{42}; lower curves give the cross-diagonal terms.
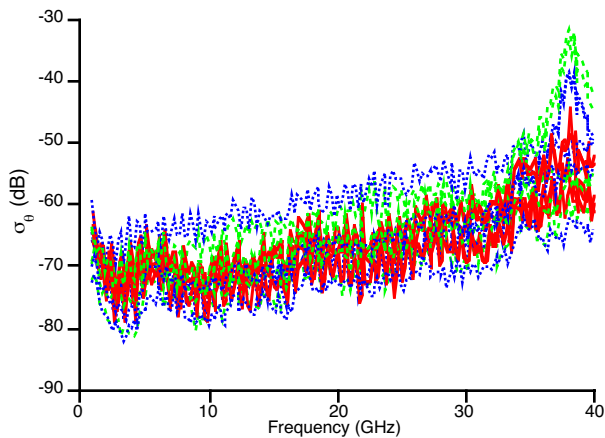

Fig. 4. Uncertainty in the correction parameters represented as the standard deviation of the magnitude for each of the 15 terms in $\boldsymbol{\theta}_{est}$.

IV. CONCLUSIONS

This paper has presented a new statistical VNA calibration method based on repeated measurements of multiple transmission-line and reflection standards. By applying a convenient least-squares optimization process, the method estimates the 15 unique correction coefficients, *plus* an optimized propagation-constant estimate for the transmission-line parameter. We demonstrated, for the first time, an application of this method to a commercial nonlinear vector network analyzer, and derived estimates of the uncertainty in the model parameters.

These statistical methods provide important insight into the quality of RF network measurements to both calibration laboratories and research and development teams, as well as limitations in a given calibration kit. They also offer important improvements in the methods of correcting VNA errors.

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